CHAPTER THREE

SURDS

When a square roots sign is squared or raised to the second power, the square root sign disappears.

(1) $(\sqrt{2})^2 = 2$ (2) $(\sqrt{3})^2 = 3$

(3) $(\sqrt{a})^2 = a$ (4) $(\sqrt{b})^2 = b$

(5) $2(\sqrt{3})^2 = 2\ (3) = 6$

(6) $5(\sqrt{2})^2 = 5\ (2) = 10$

(7) $2(\sqrt{a})^2 = 2\ (a) = 2a$

(8) $(2\sqrt{3})^2 = 2^2\ (\sqrt{3})^2 = 4\ (3) = 12$

(9) $(5\sqrt{2})^2 = 5^2\ (\sqrt{2})^2 = 25\ (2) = 50$

(10) $(a\sqrt{b})^2 = a^2\ (\sqrt{b})^2 = a^2\ (b) = a^2b$

The Perfect Squares:

The Perfect squares are:

(1) 2 x 2 = 4 (2) 3 x 3 = 9

(3) 4 x 4 = 16 (4) 5 x 5 = 25

(5) 6 x 6 = 36 (6) 7 x 7 = 49

(7) 8 x 8 = 64 (8) 9 x 9 = 81

(9) 10 x 10 = 100

-- In surd manipulation, a number which is a multiple of a perfect square must be converted into the multiple of that perfect square

Examples:

1. $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2 \times \sqrt{2} = 2\sqrt{2}$.

2. $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \text{ x } \sqrt{3} = 2 \text{ x } \sqrt{3} = 2\sqrt{3}$.

3. $\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \text{ x } \sqrt{2}$
$= 4 \text{ x } \sqrt{2} = 4\sqrt{2}$

4. $\sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \text{ x } \sqrt{3} = 3\sqrt{3}$

5. $\sqrt{69} = \sqrt{9 \times 7} = \sqrt{9} \text{ x } \sqrt{7} = 3\sqrt{7}$

6. $\sqrt{80} = \sqrt{16 \times 5} = \sqrt{16} \text{ x } \sqrt{5} = 4\sqrt{5}$

(7) $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \text{ x } \sqrt{2} = 5\sqrt{2}$

(8) $\sqrt{147} = \sqrt{49 \times 3} = \sqrt{49} \text{ x } \sqrt{3} = 7\sqrt{3}$

(9) $\sqrt{125} = \sqrt{25 \times 3} = \sqrt{25} \text{ x } \sqrt{3} = 5\sqrt{3}$

<u>How to determine whether a number is a multiple of a perfect square:</u>

-- The perfect squares 4, 9, 16, 25, 36, 49, 64, 81 and 100 are what we shall make use of.

-- When a number is given and we want to know whether it is a multiple of a perfect square, we start dividing the numbers with the perfect squares in turn, starting with the highest which is 100.

-- If we get an answer which is a whole number but not a decimal, then that particular number is a multiple of a perfect square.

-- Far example if we are given $\sqrt{448}$ and we want to know whether or not we can break it down, we first divide 448 by 100 which gives us 4.48.

-- Since this is a decimal, then we try the next perfect square which is 81.

-- Dividing 448 by 81 gives us 5.5 which is also a decimal. We then try the next perfect square which is 64.

-- Dividing 448 by 64 gives us 7, which is a whole numbers.

- This implies that $\sqrt{448}$
$= \sqrt{64 \times 7} = \sqrt{64} \text{ x } \sqrt{7} = 8\sqrt{7}$.

Example (2) Let us now determine whether $\sqrt{294}$ can be simplified or broken down.

- Dividing 294 by 100 gives us 2.94 which is a decimal.

- Dividing 294 by 81 gives us 3.6 which is a decimal.

- Diving 294 by 64 gives us 4.59 which is also a decimal.

- Dividing 294 by 49 gives us 6 which is a whole number

- This implies that √294
  $= \sqrt{49 \times 6}$ = √49 x √6 = 7√6.

Examples (3) Now let us determine whether √150 can be simplified or broken down.

- Dividing 150 by 100 gives us 1.50 which is a decimal

- Dividing 150 by 81 gives us 1.85 which is a decimal.

- Dividing 150 by 49 gives us 3.06 which is a decimal.

- Dividing 150 by 36 gives us 4.2 which is a decimal.

- Dividing 150 by 25 gives us 6 which is a whole number.

- This implies that √150
  $= \sqrt{25 \times 6}$ = √25 x √6 = 5√6.

NB: If we divide a given number by all the perfect Squares we are supposed to use, and in each case get a decimal, then that number cannot be simplified or broken down.

For example, assume that we want to know whether or not √271 can be simplified or broken down.

-- Dividing 271 by 100 gives us 2.71 which is a decimal.

-- Dividing 271 by 81 gives us 3.34 which is a decimal

- Dividing 271 by 64 gives us 4.2 which is decimal.
- Dividing 271 by 49 gives us 5.5 which is decimal.

-- Dividing 271 by 36 gives us 7.5 which is a decimal.

-- Dividing 271 by 25 gives us 10.8 which is decimal.

-- Dividing 271 by 16 gives us 16.9 which is decimal.
-- Dividing 271 by 9 gives us 30.1 which is a decimal.

-- Dividing 271 by 4 gives us 67.7 which is also a decimal.

Since when all the perfect squares concerned, when used to divide the given number gave us decimals as our answer, then $\sqrt{271}$ cannot be simplified and should be left as $\sqrt{271}$.

Addition of surds:

-- In surd addition, we can only add if the numbers under the square root signs are the same.

-- If they are not the same, then nothing can be done.

Examples

(1) $a\sqrt{b} + c\sqrt{b} = (a + c)\sqrt{b}$

(2) $5\sqrt{2} + 4\sqrt{2} = (5 + 4)\sqrt{2} = 9\sqrt{2}$

(3) $6\sqrt{7} + 2\sqrt{7} = (6 + 2)\sqrt{7} = 8\sqrt{7}$

(4) $3\sqrt{5} + 2\sqrt{5} = (3 + 2)\sqrt{5} = 5\sqrt{5}$

(5) $2\sqrt{7} + 3\sqrt{7} = (2 + 3)\sqrt{7} = 5\sqrt{7}$

(6) $2\sqrt{3} + \sqrt{3} = 2\sqrt{3} + 1\sqrt{3} = (2 + 1)\sqrt{3} = 3\sqrt{3}$

(7) $5\sqrt{2} + \sqrt{2} = 5\sqrt{2} + 1\sqrt{2} = (5 + 1)\sqrt{2} = 6\sqrt{2}$

(8) $2\sqrt{5} + 4\sqrt{5} + 3\sqrt{5} = (2 + 4 + 3)\sqrt{5} = 9\sqrt{5}$

(9) $5\sqrt{2} + 2\sqrt{2} + 4\sqrt{2} = (5 + 2 + 4)\sqrt{2} = 11\sqrt{2}$

(10) $\sqrt{3} + 4\sqrt{3} + 6\sqrt{3} = 1\sqrt{3} + 4\sqrt{3} + 6\sqrt{3} = (1 + 4 + 6)\sqrt{3} = 11\sqrt{3}$

(11) $2\sqrt{3} + 4\sqrt{2} = 2\sqrt{3} + 4\sqrt{2}$

(12) $5\sqrt{7} + 2\sqrt{6} = 5\sqrt{7} + 2\sqrt{6}$

(13) $4\sqrt{2} + 3\sqrt{5} = 4\sqrt{2} + 3\sqrt{5}$

(14) $5\sqrt{2} + 3\sqrt{3} + 2\sqrt{7} = 5\sqrt{2} + 3\sqrt{3} + 2\sqrt{7}$

.

Simplify each of the following:

(Q1) $2 + 3\sqrt{5} + 6 + 4\sqrt{5}$

Solution

$2 + 3\sqrt{5} + 6 + 4\sqrt{5} = 2 + 6 + 3\sqrt{5} + 4\sqrt{5}$

$= 8 + (3 + 4)\sqrt{5} = 8 + 7\sqrt{5}$

(Q2) $6\sqrt{7} + 1 + 4\sqrt{7} + 3 + 2\sqrt{7}$

Solution

$6\sqrt{7} + 1 + 4\sqrt{7} + 3 + 2\sqrt{7} = 6\sqrt{7} + 4\sqrt{7} + 2\sqrt{7} +1 +3$

$= (6 + 4 + 2) \sqrt{7} + 4 = 12\sqrt{7} + 4$

(Q3) $2\sqrt{3} + 5\sqrt{7} + 5\sqrt{3} + 2$

Solution

$2\sqrt{3} + 5\sqrt{7} + 5\sqrt{3} + 2 = 2\sqrt{3} + 5\sqrt{3} + 5\sqrt{7} + 2$

$= (2 + 5) \sqrt{3} + 5\sqrt{7} + 2 = 7\sqrt{3} + 5\sqrt{7} + 2$

(Q4) $2 + 5\sqrt{2} + \sqrt{3} + 6\sqrt{2} + 7\sqrt{3} + 6$

Solution

$2 + 5\sqrt{2} + \sqrt{3} + 6\sqrt{2} + 7\sqrt{3} + 6 = 2 + 5\sqrt{2} + 1\sqrt{3} +6\sqrt{2} + 7\sqrt{3} + 6$

$= 2 + 6 + 5\sqrt{2} + 6\sqrt{2} + 1\sqrt{3} +7\sqrt{3}$

$= 8 + (5 + 6) \sqrt{2} + (1 + 7) \sqrt{3}$

$= 8 + 11\sqrt{2} + 8\sqrt{3}$

(Q5) $5^1/_2 + 3\sqrt{2} + 6\sqrt{2} + 7\sqrt{3} + {}^1/_2$

Solution

$5^1/_2 + 3\sqrt{2} + 6\sqrt{2} + 7\sqrt{3} + {}^1/_2$

$= 5^1/_2 + {}^1/_2 + 3\sqrt{2} + 6\sqrt{2} + 7\sqrt{3}$

$= 6 + (3 + 6) \sqrt{2} + 7\sqrt{3} = 6 + 9\sqrt{2} + 7\sqrt{3}$

(Q6) $2\sqrt{8} + 3\sqrt{3} + 1$

Solution

$2\sqrt{8} + 3\sqrt{3} + 1 = 2\sqrt{4 \times 2} + 3\sqrt{3} + 1$

$= 2x\sqrt{4}\ x\ \sqrt{2} + 3\sqrt{3} + 1$

$= 2\ x\ 2\ x\ \sqrt{2} + 3\sqrt{3} + 1 = 4\sqrt{2} + 3\sqrt{3} + 1$

(Q7) $3\sqrt{2} + 2 + 2\sqrt{8} + 4\sqrt{2} + 6$

Solution

$3\sqrt{2} + 2 + 2\sqrt{8} + 4\sqrt{2} + 6$

$= 3\sqrt{2} + 2 + 2\sqrt{4 \times 2} + 4\sqrt{2} + 6$

$= 3\sqrt{2} + 2 + 2x\sqrt{4}x\sqrt{2} + 4\sqrt{2} + 6$

$= 3\sqrt{2} + 2 + 2\ x\ 2\ x\sqrt{2} + 4\sqrt{2} + 6$

$= 3\sqrt{2} + 4\sqrt{2} + 4\sqrt{2} + 6 + 2$

$= (3 + 4 + 4) \sqrt{2} + 8$

$=11\sqrt{2} + 8$

(Q8) $5+3\sqrt{27} + 2 + 6\sqrt{3} + 2\sqrt{2} + \sqrt{12} + 1$

Solution

$5+3\sqrt{27} + 2 + 6\sqrt{3} + 2\sqrt{2} + \sqrt{12} + 1$

$= 5 + 3\sqrt{27} + 2 + 6\sqrt{3} + 2\sqrt{2} + \sqrt{12} + 1$

$= 5 + 3x\sqrt{9}x\sqrt{3} + 2 + 6\sqrt{3} + 2\sqrt{2} + \sqrt{4}x\sqrt{3} +1$

$= 5 + 3x3x\sqrt{3} + 2 + 6\sqrt{3} + 2\sqrt{2} + 2x\sqrt{3} + 1$

$= 5 + 9\sqrt{3} + 2 + 6\sqrt{3} + 2\sqrt{2} + 2\sqrt{3} + 1$
$= 5 + 2 + 1 + 9\sqrt{3} + 6\sqrt{3} + 2\sqrt{3} + 2\sqrt{2}$
$= 8 + (9 + 6 + 2)\sqrt{3} + 2\sqrt{2}$
$= 8 + 17\sqrt{3} + 2\sqrt{2}$

(Q9) $4 + 2\sqrt{32} + 3\sqrt{2} + 1 + 2\sqrt{50}$

Solution

$4 + 2\sqrt{32} + 3\sqrt{2} + 1 + 2\sqrt{50}$
$= 4 + 2\sqrt{16 \times 2} + 3\sqrt{2} + 1 + 2\sqrt{25 \times 2}$
$= 4 + 2 x \sqrt{16}\ x\ \sqrt{2} + 3\sqrt{2} + 1 + 2 x \sqrt{25}\ x\ \sqrt{2}$
$= 4 + 2\ x\ 4\ x\ \sqrt{2} + 3\sqrt{2} + 1 + 2\ x\ 5\ x\ \sqrt{2}$
$= 4 + 8\sqrt{2} + 3\sqrt{2} + 1 + 10\sqrt{2}$
$= 4 + 1 + 8\sqrt{2} + 3\sqrt{2} + 10\sqrt{2}$
$= 5 + (8 + 3 + 10)\sqrt{2}$
$= 5 + 21\sqrt{2}$

(Q10) $3\sqrt{7} + 5 + 2\sqrt{7} + 3\sqrt{16} + 2\sqrt{25} + 4\sqrt{128}$

Solution

$3\sqrt{7} + 5 + 2\sqrt{7} + 3\sqrt{16} + 2\sqrt{25} + 4\sqrt{128}$
$= 3\sqrt{7} + 5 + 2\sqrt{7} + 3\ (4) + 2\ (5) + 4\sqrt{64 \times 2}$
$= 3\sqrt{7} + 5 + 2\sqrt{7} + 12 + 10 + 4 x \sqrt{64}\ x\ \sqrt{2}$
$= 3\sqrt{7} + 2\sqrt{7} + 5 + 12 + 10 + 4\ x\ 8\ x\ \sqrt{2}$
$= (3 + 2)\sqrt{7} + 27 + 32\sqrt{2}$
$= 5\sqrt{7} + 27 + 32\sqrt{2} = 5\sqrt{7} + 32\sqrt{2} + 27$

NB: You must first check whether 128 is a multiple of a perfect square or not.